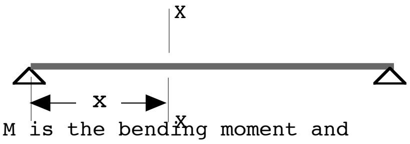
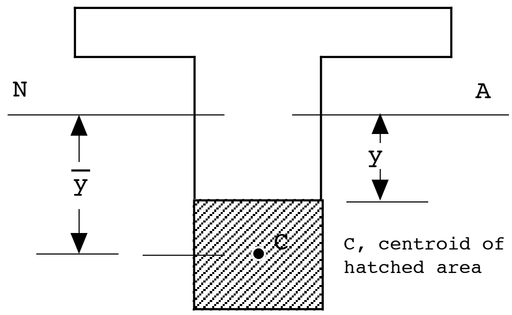
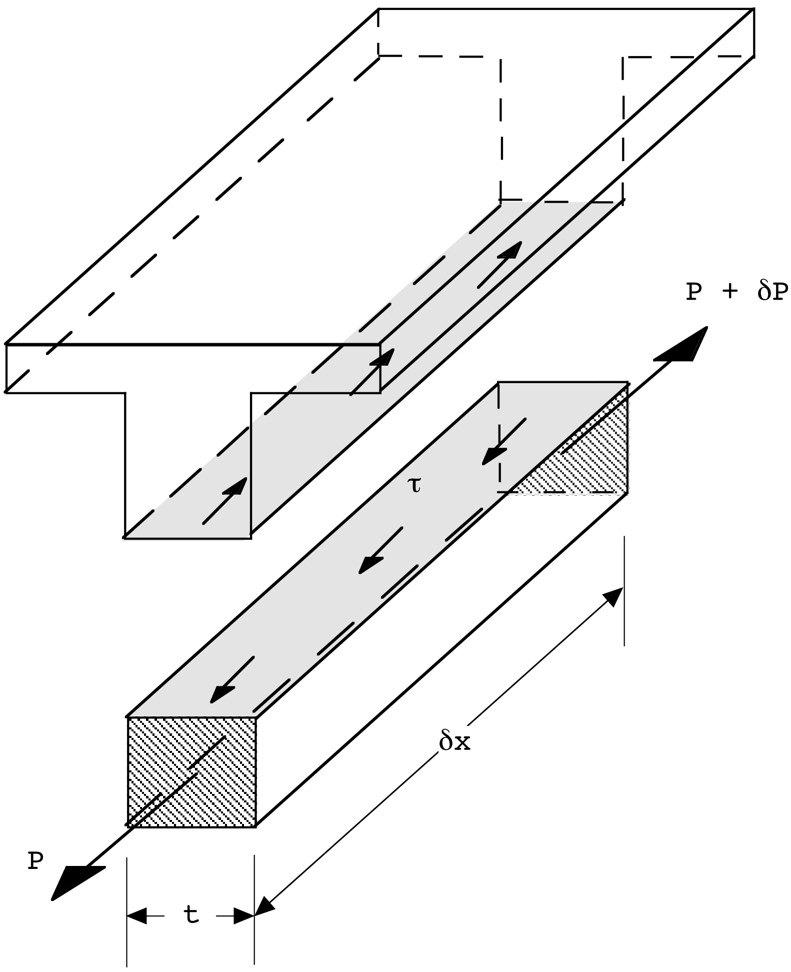
Shear stress in beams



M is the bending moment and
F is the shear force at X-X



X-X



Let σ be the normal bending stress then $P = \int \sigma A \, dA$ where $\sigma = My/I$.

I is the second moment of area of the beam about the neutral axis.

Therefore $P = \int MyIA \, dA = MA\bar{y}/I$ where A is the hatched area above.

Longitudinal equilibrium gives: $\delta P = \tau(t \, \delta x)$

Therefore $\tau t = dPdx = FA\bar{y}/I$ or

$$\tau = FA\bar{y}It$$ provided that the beam is uniform.

Shear Stress Distribution in Beams

I. Thick-walled beams

Qu.1
The channel section shown in Fig.1 is simmply-supported over a span of 5m and carries a UDL of intensity 15kN/m over its entire length. Sketch the shear stress distribution diagram at the point of maximum shearing force, and give important values. Determine the ratio of maximum shear stress to average shear stress.

Answer. 3, 9.2, 9.3 N/mm^2 ; 2.42

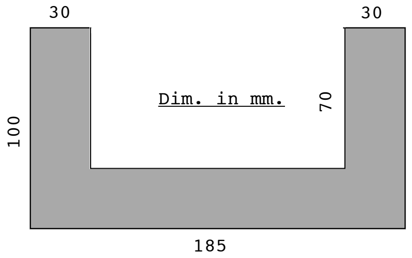


Fig. 1

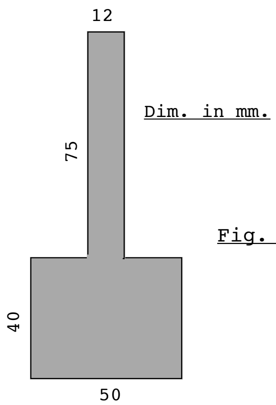


Fig. 2

Qu.2
Fig.2 shows the cross-section of a beam that carries a shear force of 20kN. Determine the shear stress distribution.

Answer. 21.7, 5.2, 5.23 N/mm^2 ; 2.42

Qu.3
The cross-section of a beam is an isoceles triangle of base B and height H, the base being arranged in a horizontal plane. Find the shear stress at the neutral axis due to a shear force Q acting on the cross-section and express it in terms of the average shear stress.

Answer. 8Q/(3BH) , $4\tau_{avg}/3$

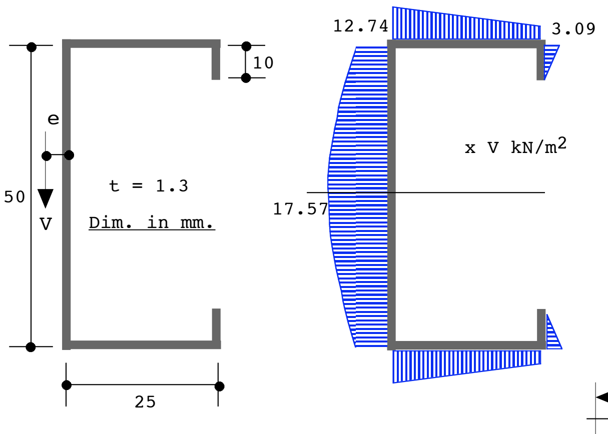
II. Thin-walled sections

Qu.4

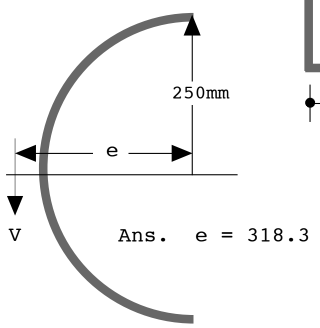
A beam having the cross-section shown in Fig.3 is made of metal having constant wall thickness of 1.3mm. Throught what point must the applied vertical load pass in order that there shall be no twisting of the section? Sketch the shear stress distribution.

Qu.5

Determine the position of the shear centre, e, of the beams shown in Figs.4-6

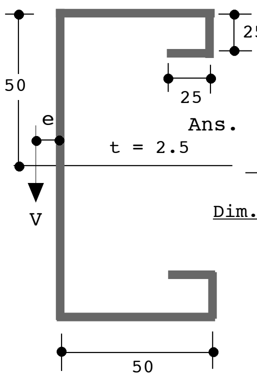


Ans. e = 13.9 Fig.3



Ans. e = 318.3

Fig.4



Ans. e = 37.1

Fig.5

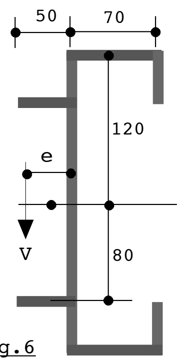
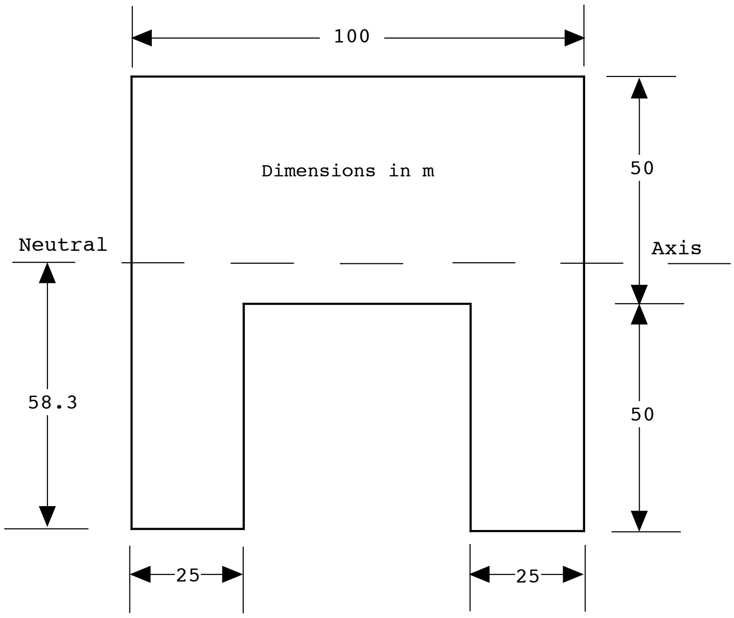


Fig.6

Example 1 Draw the shear stress distribution. Take F = 400kN



Calculate the following:

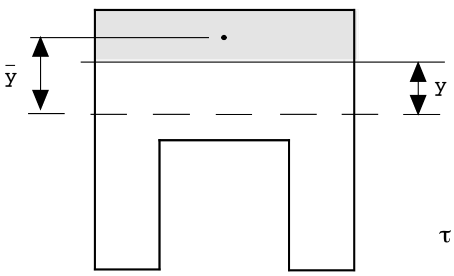
1. A = 7500 mm^2

2. $\bar{y}$ = 58.3 mm

3. I_{NA} = 5.729 x 10^6 mm^4

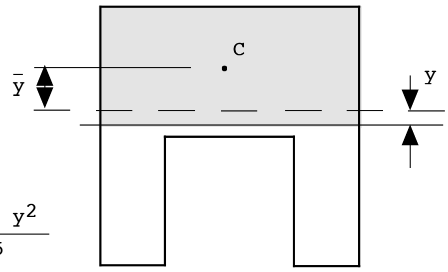
Use $\tau = \dfrac{FA\bar{y}}{It}$

Case 1

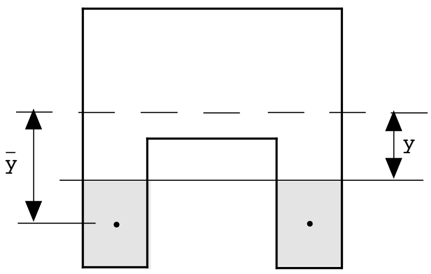


$$\tau = \frac{1738.9 - y^2}{28.645}$$

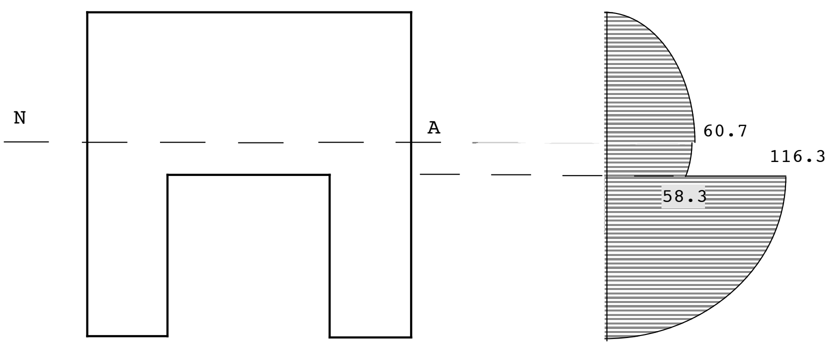
Case 2



Case 3

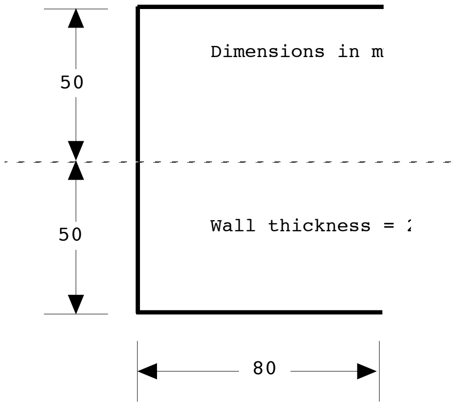


$$\tau = \frac{3398.9 - y^2}{28.645}$$



Shear stress distribution in N/mm^2

Example 2 Draw the shear flow distribution. Take F = 10kN. Also find th location of the shear centre of the thin-walled channel.



Calculate the following:

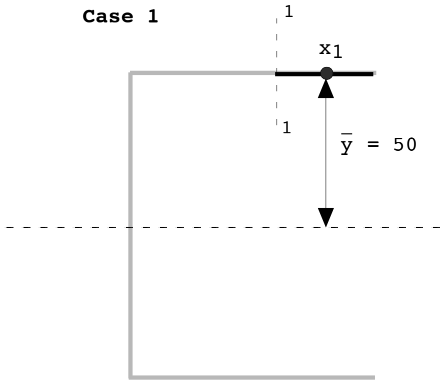
1. $A = 520\ mm^2$

2. $y = 50\ mm$

3. $I_{NA} = 966.7 \times 10^3\ mm^4$

Use $f = \dfrac{FA\bar{y}}{I}$

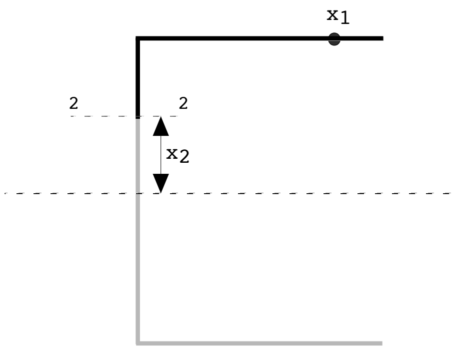
Case 1



$A_1 = 2\ x_1$

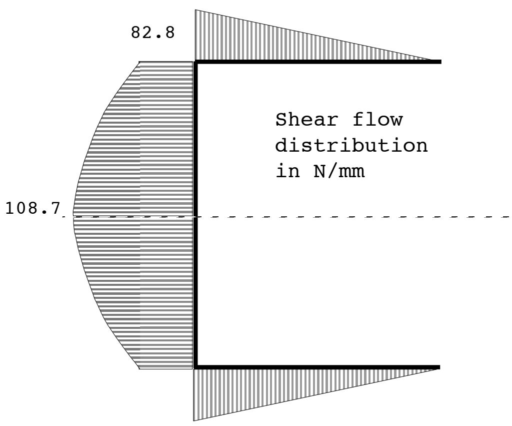
$f_1 = 1.034\ x_1$ N/mm

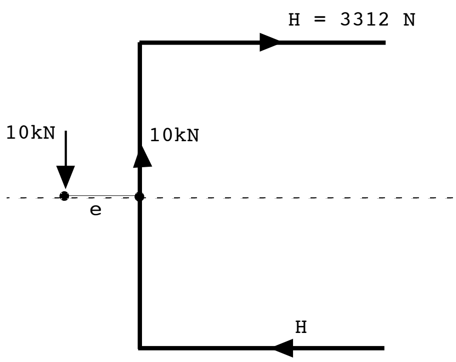
Case 2



$$A_2 = \underbrace{160}_{y\,=\,50} + \underbrace{2\,(50 - x_2)}_{y\,=\,(50\,+\,x_2)/2}$$

$$f_2 = \frac{10000\ (8000 + 50^2 - x_2^2)}{966.7 \times 10^3}$$
$$= 82.8 + 1.034\ (50^2 - x_2^2)/100$$





$10000\ e = H \times 100$

<u>$e = 33.1$mm</u>