Continuous Beams - Flexibility Method

Qu.1

Sketch the BM diagram for the beam shown in Fig.1. Take E = 200kN/mm^2.

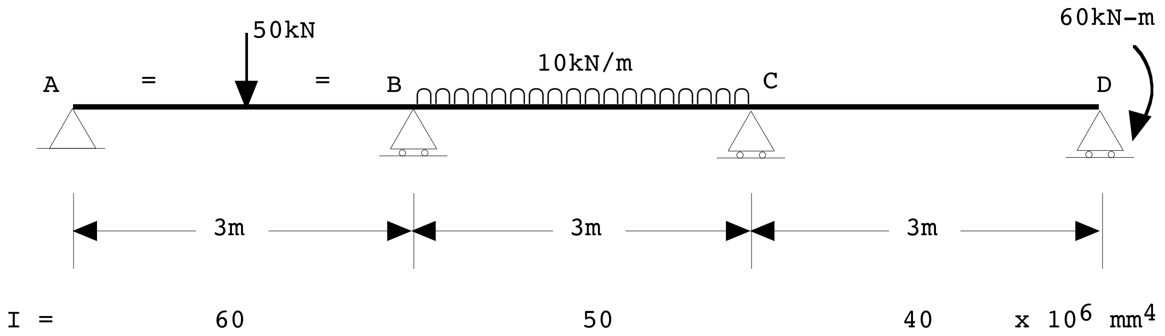
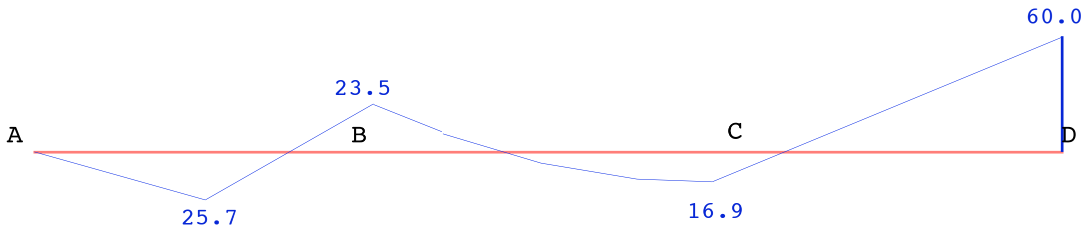


Fig.1



BM diagram in kN-m units

Qu.2

Sketch the BM diagram for the beam shown in Fig.2. Take EI = constant.

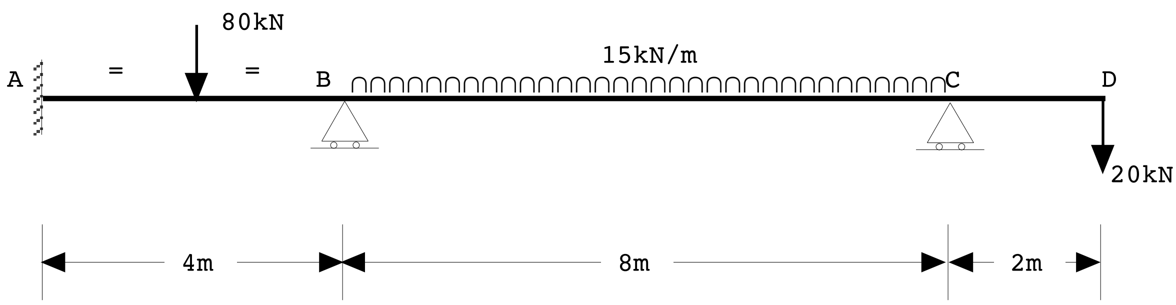
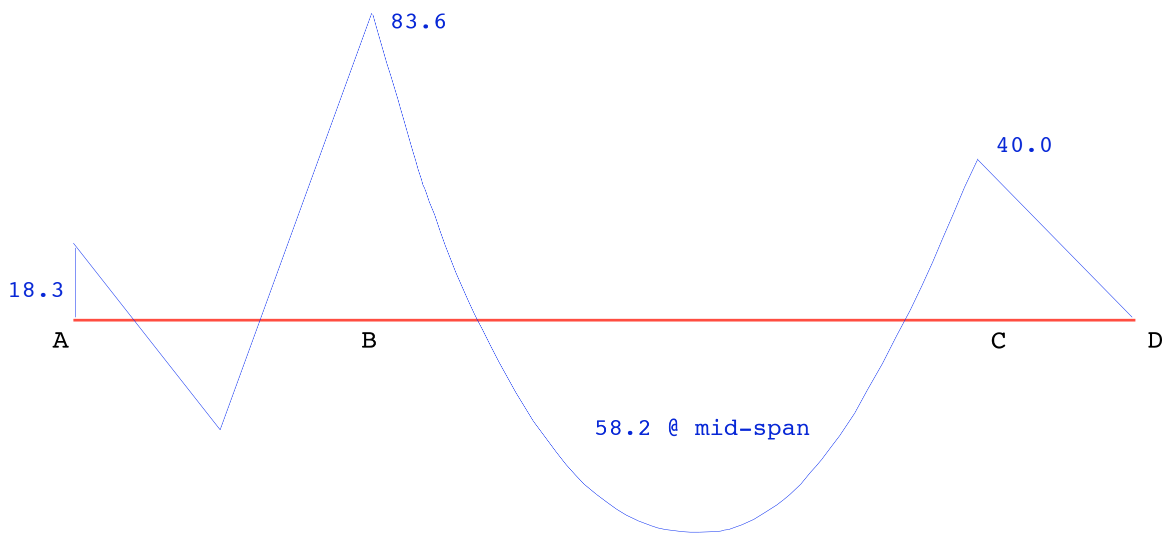


Fig.2



BM diagram in kN-m units

Qu.3

Sketch the BM diagram for the beam shown in Fig.3. Also calculate the support reactions. Take EI const. [R_A = -29.9 , R_B = 236.8, R_C = 381.1kN]

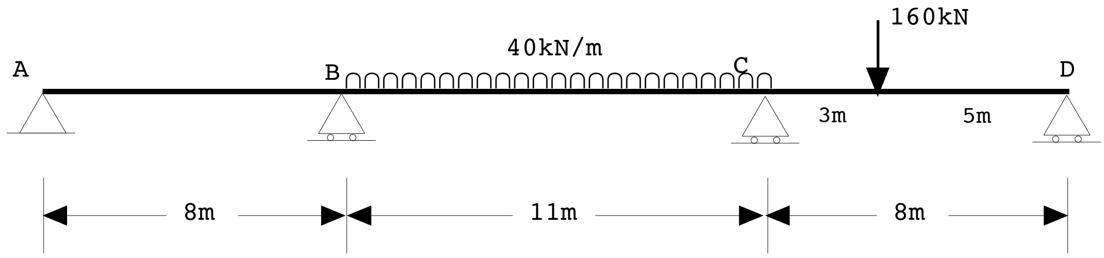
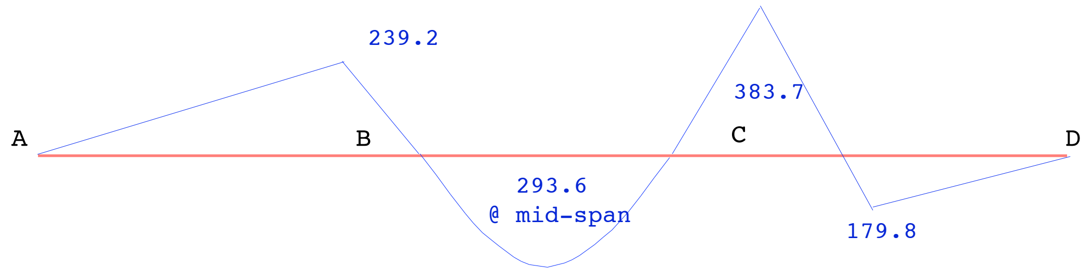


Fig.3



BM diagram in kN-m units

Qu.4

Sketch the BM diagram for the beam shown in Fig.4. Support B undergoes a settlement of 15mm. Take E = 200kN/mm^2 and I = 1 x 10^{-4}m^4.

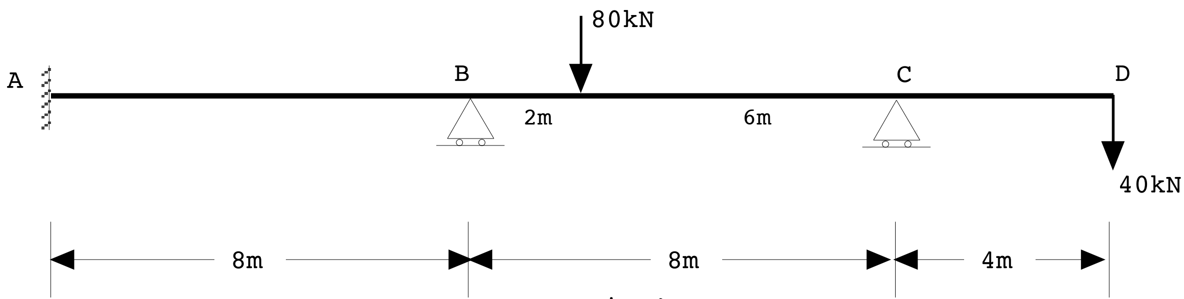
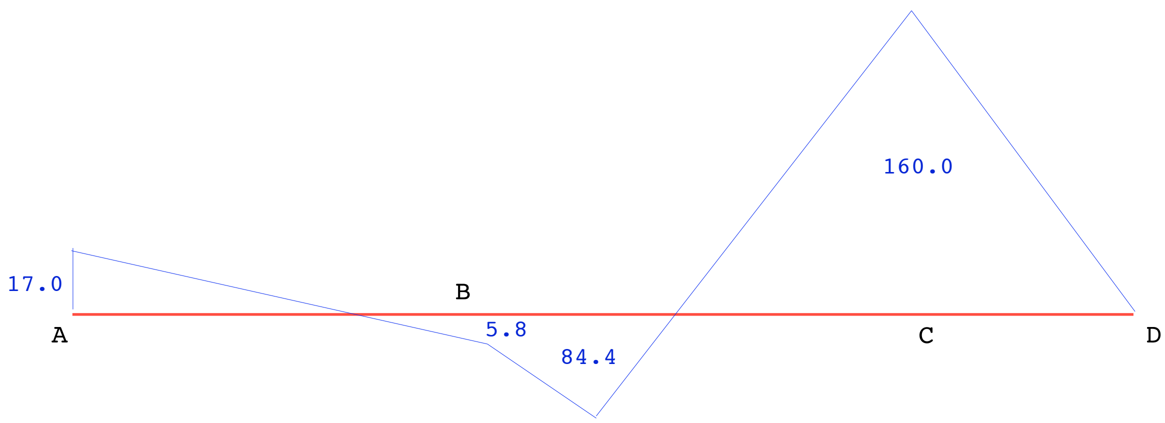


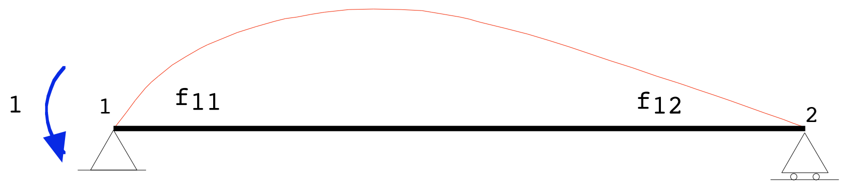
Fig.4



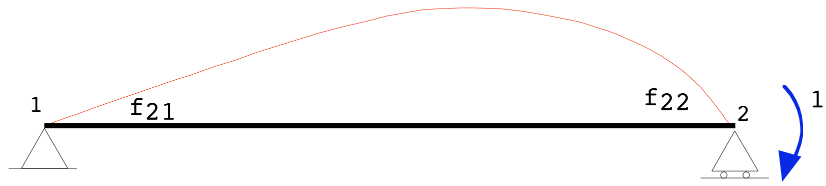
BM diagram in kN-m units

End rotations of simply supported beams

Span = L   Flexural rigidity = EI

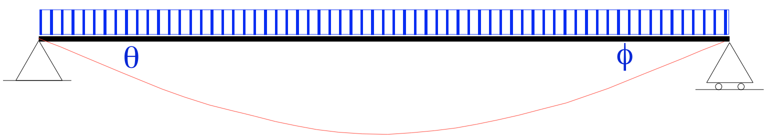


$$f_{11} = \frac{L}{3EI} \qquad f_{12} = \frac{L}{6EI}$$

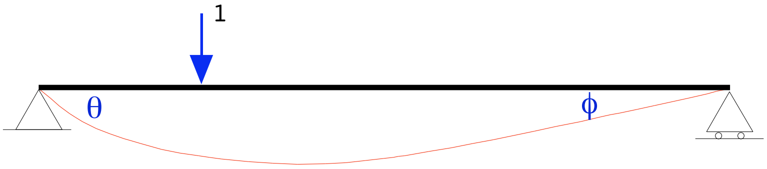


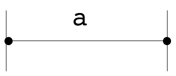
$$f_{21} = \frac{L}{6EI} \qquad f_{22} = \frac{L}{3EI}$$

UDL of intensity 1



$$\theta = \phi = \frac{L^3}{24EI}$$



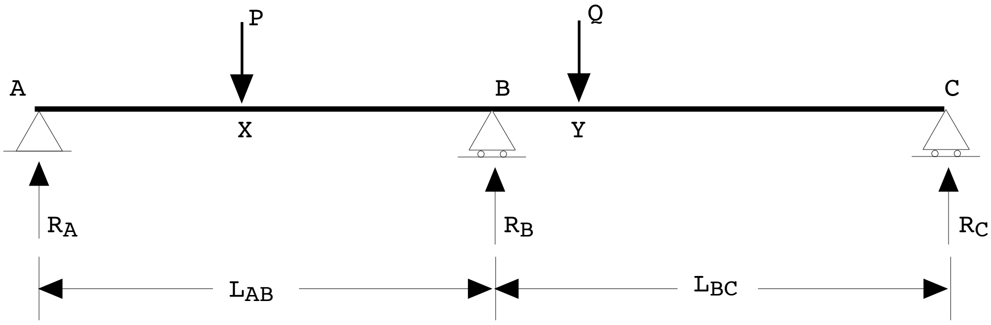


$$\alpha = \frac{a}{L}$$

$$\theta = \frac{L^2(1 - \alpha)(2 - \alpha)\alpha}{6EI} \qquad \phi = \frac{L^2(1 - \alpha^2)\alpha}{6EI}$$

Flexibility method applied to continuous beams

Example Two-span continuous beam with the flexural rigidity EI = constant



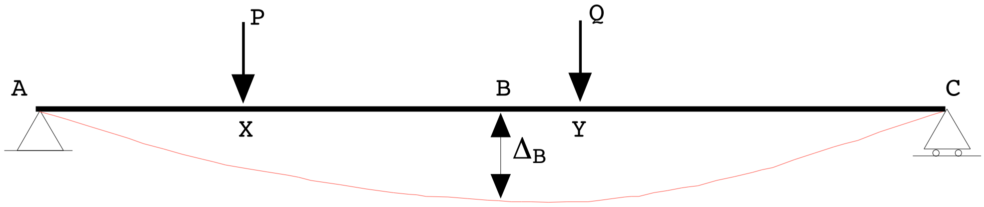
Using statics

Res. vertically: $R_A + R_B + R_C = P + Q$ (1)

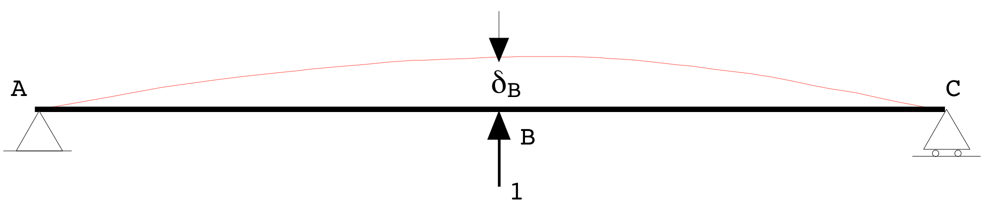
Taking mom. about C: $R_A \times (L_{AB} + L_{BC}) + R_B \times L_{BC} = P \times XC + Q \times YC$ (2)

The three reactions are the unknowns but only two equations are available. Therefore the problem is statically indeterminate. We must use compatibility of displacements to generate an additional equation in order to complete the solution.

One procedure for doing this is illustrated in the figures below:



Step 1 Remove prop and calculate the deflection Δ_B

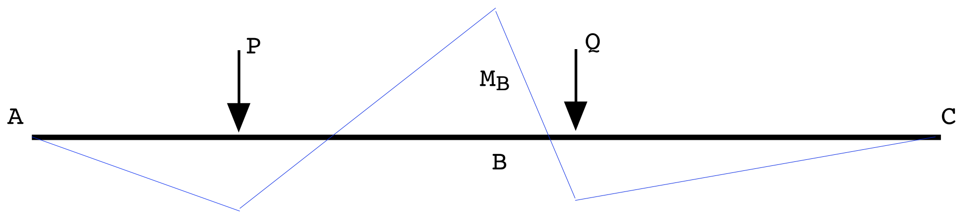


Step 2 Introduce unit force at B and calculate δ_B

For compatibility $R_B = \Delta_B / \delta_B$ (3)

We use these three equations to solve for R_A, R_B and R_C respectively.

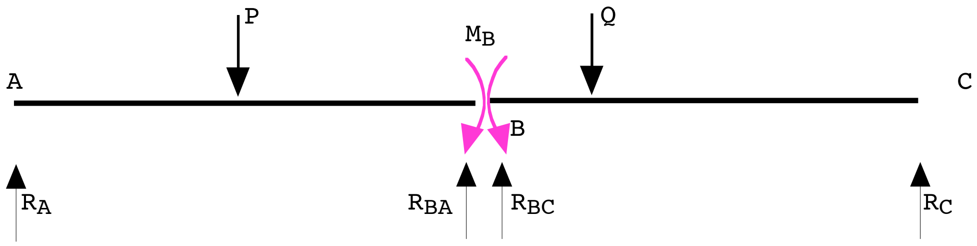
The bending moment at B is $M_B = R_A \times L_{AB} - P \times XB$ (4)



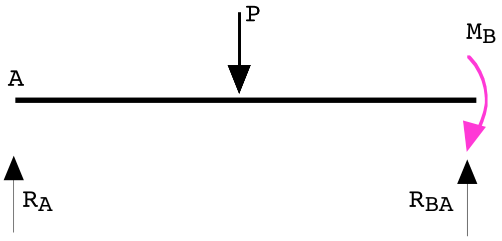
Bending Moment Diagram

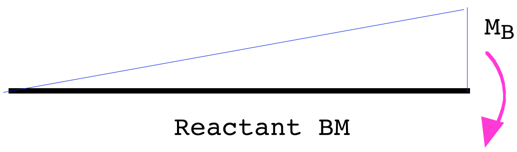
The major disadvantage of this method arises when EI varies from span to span. The deflections cannot be calculated from standard tables which assume that the flexural rigidity EI is constant.

The free-body diagram for the individual spans are given below:

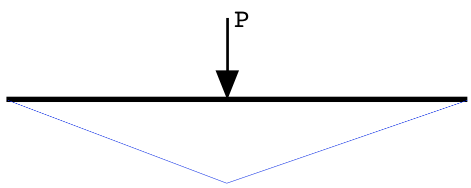


$R_B = R_{BA} + R_{BC}$





Reactant BM



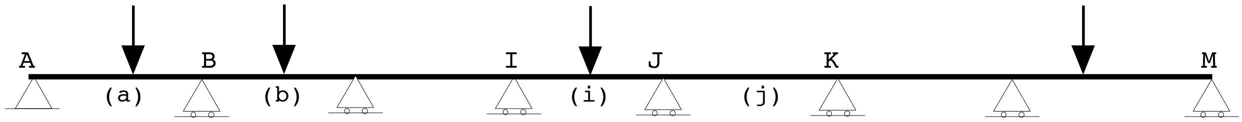
Free BM

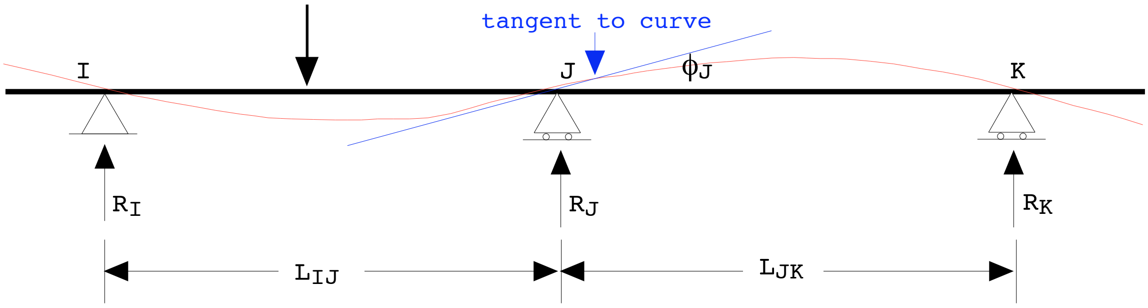
Final BM = Reactant BM + Free BM

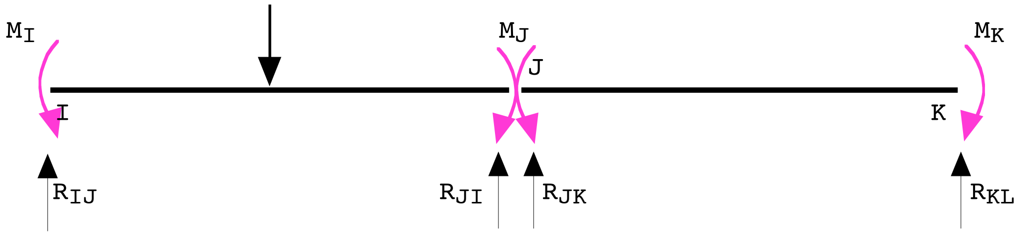
A powerful method that does not suffer from this defect is now described. It uses displacement compatibility and leads to the Three Moment Theorem. The method breaks a continuous beam into a series of simply supported spans. The slopes are therefore discontinuous over the supports. Rotations are then introduced via the reactant moments to 'heal the cuts'.

Three Moment Theorem

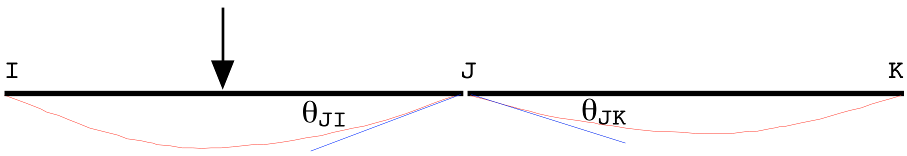
Consider a continuous beam consisting of N spans.



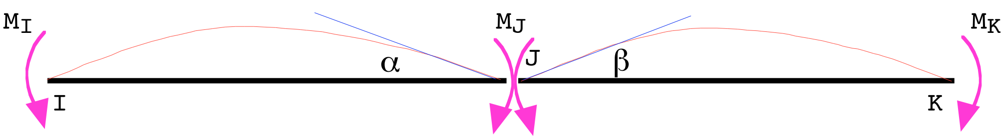




$R_J = R_{JI} + R_{Jk}$



<u>Stage A</u> : End rotations due to applied loads



<u>Stage A</u> : End rotations due to reactant moments

In order to restore continuity at joint J,

$$\phi_J = \theta_{JI} - \alpha = \beta - \theta_{JK}$$

Flexibility Coefficients: End rotations due to unit couples

Making use of the flexibility coef's shown above we may write the compatibility condition as follows:

$$\theta_{JI} - M_I\, f^i{}_{12} - M_J\, f^i{}_{22} = M_J\, f^i{}_{11} - M_K\, f^i{}_{21} - \theta_{JK}$$

or

$$\boxed{M_I\, f^i{}_{12} + M_J\, (f^i{}_{11} + f^i{}_{22}) + M_K\, f^i{}_{21} = \theta_{JI} + \theta_{JK}}$$

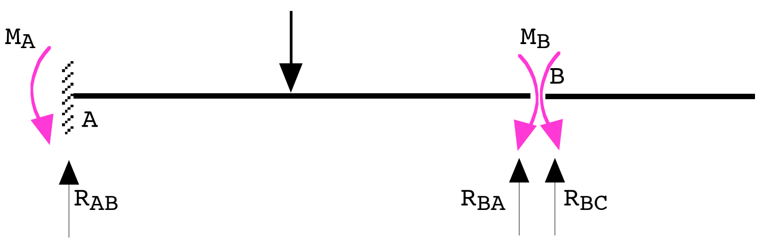
This is the so-called Three Moment Theorem (due to Clapeyron).

Settlement of supports

The theorem can be easily extended to deal with settlement of supports. Let Δ_J be the settlement of support J. The rotations on each side of support J due to the differential settlement are $(\Delta_J - \Delta_I)/L_{IJ}$ and $(\Delta_J - \Delta_K)/L_{JK}$ respectively. We get

$$\boxed{M_I\, f^i{}_{12} + M_J\, (f^i{}_{11} + f^i{}_{22}) + M_K\, f^i{}_{21} = (\theta_{JI} - (\Delta_J - \Delta_I)/L_{IJ}) + (\theta_{JK} - (\Delta_J - \Delta_K)/L_{JK})}$$

Fixed end (zero rotation)



A fixed end, A in the figure above, does not rotate. The rotation θ_{AB} must therefore be balanced by the reactant rotations. We have

$$\boxed{M_A\, f^a{}_{11} + M_B\, f^a{}_{21} = \theta_{AB} - (\Delta_A - \Delta_B)/L_{AB}}$$